Abstract - The conventional CAD modeling approaches only provide geometry and topology of the object but do not contain any information with regard to the materials of the object and so can not be used for the fabrication of heterogeneous objects (HO) through rapid prototyping. A CAD modeling approach using voxelization based material model is represented for the fabrication of HO with simple object geometries and complex material variations. In this framework, volumetric dataset is employed to represent the material variations, which helps in flexible manipulability to HO representation, while geometry model is used to describe the shape of an object, which guarantees the accuracy of fabricated heterogeneous objects. The relation between basic requirements along with respective solutions for a standard CAD modeling approach is established. Further a framework for developing the required CAD model for rapid prototype (RP) machines to fabricate heterogeneous objects is designed. 

Keywords: Heterogeneous Object, Rapid Prototyping, Voxelization

I. INTRODUCTION

The term ‘heterogeneous object’ is defined such that it can have different material composition within an object. There are three subclasses of heterogeneous object e.g.[12]:
- Multiple materials object
- Object with sub-objects embedded
- Object without clear material boundary (Functionally graded materials, FGM)

Traditional CAD systems, used for conventional design method, can only represent the geometry and topology of an object. No material information is available within the representation which is required for heterogeneous objects. With the capability to fabricate heterogeneous objects, functionally efficient and cost reducing designs can be realized. Rapid prototyping (RP) techniques allow heterogeneous material objects to be produced using 3D CAD models by varying material composition region-wise, layer-wise, or point-wise. The required 3D CAD model should have not only the geometric information but also the information of material, property, etc. at each point inside an object. In order to take full advantage of the greatest potential of heterogeneous objects, one must have matching capabilities for their computer modeling, analysis and design optimization. The primary focus of the recent research development in these fields is on the computer representation schemes for heterogeneous objects, by extending the mathematical models and computer data structures of the modern solid modeling techniques to include discrete material regions of interfacial boundaries and heterogeneous properties.

Recent studies show that an effective heterogeneous CAD modeling system should at least meet the following specifications e.g. [4]:
- Intuitive in representing geometry, topology and material information simultaneously
- Capable of representing complex solids: the solids to be modeled may be complex in geometry as well as in material variations
- Compact and exact: the representation should be compact, and both the geometry and material information can be retrieved accurately and efficiently
- The representation of material properties must be compatible with current or proposed standards for geometric modeling representations as described in ISO 10033. This is essential to exchange data among design, analysis and manufacturing process plan domains.

This paper is organized as follows: in section 2, the previous work is reviewed; Section 3 is brief representation of CAD model with information flow for fabrication and processing of HO; the relation between requirements and the respective solutions is established in section 4; Section 5 represents developed mathematical model for the fabrication of heterogeneous objects; a framework for the CAD modeling approach is introduced in section 6 and the final section is conclusion with future scope to extend this work.

II. REVIEW OF PREVIOUS RESEARCH

Approaches of modeling of HO have been extensively studied in computer and manufacturing community. Kumar and Dutta proposed an approach to model multimaterial objects based on R-m sets and R-m classes primarily for application in layered manufacturing. Boolean operators were defined to facilitate the modeling process e.g. [5-6]. Jackson et al. proposed a local composition control (LCC) approach to represent heterogeneous object in which a mesh model is divided into tetrahedrons and different material compositions are evaluated on the nodes of the tetrahedrons by using Bernstein polynomials e.g. [3],[7]. Chiu developed material tree structure to store different compositions of an object e.g. [2]. The material tree was then added to a data file to construct a modified format being suitable for RP manufacturing. Siu and Tan developed a scheme named ‘source-based’ method to distribute material primitives, which can vary any material with an object e.g. [12]. The feature-based modeling scheme was extended to heterogeneous object representation through boundary conditions of a virtual diffusion problem in the solid, and then designers could use it to control the material distribution e.g. [10-11]. Liu extended his work in by taking parameterized functions in terms of distance(s) and functions using Laplace...
equation to smoothly blend various boundary conditions, through which designers could edit geometry and composition simultaneously [4],[10]. Kou and Tan suggested a hierarchical representation for heterogeneous object modeling by using B-rep to represent geometry and a heterogeneous feature tree to express the material distributions e.g. [4]. Various methods for designing and optimizing objects composed of multiple regions with continuously varying material properties have been developed. Wang and Wang proposed a level-set based variational scheme [12]. Biswas et al. presented a mesh-free approach based on the generalized Taylor series expansion of a distance field to model and analyze a heterogeneous object satisfying the prescribed material conditions on a finite collection of material features and global constraints [13-14]. However, almost all of the research interests are mainly focused on the computer representation of heterogeneous object, rather than the whole procedure for rapid prototyping fabrication of heterogeneous object. The approaches were verified in commercial software packages, such as Solidworks and Unigraphics [7],[10]. A commercial CAD package independent system is developed to deal with the HO modeling, but not including the slicing procedure for RP manufacturing [11]. In this paper, we just address the CAD modeling approach for HO. A detailed description of each module cannot be presented for the paper length.

III. MODELING AND PROCESSING OF HETEROGENEOUS OBJECTS

The RP processes are dependent on a CAD model of the heterogeneous object which generates the required information for driving the RP machine. The necessary tasks to generate this information are termed as process planning tasks. RP processes can fabricate heterogeneous objects by selectively depositing various materials in a point-wise fashion using 3D-CAD data representation without special tooling. In these processes, a uniform layer of powder is spread over the built area and the different layers are joined together by different methods to form the prototype. The information flow of modeling and processing the heterogeneous objects is shown in Fig. 1. It describes the necessity of developing material modeling system along with the geometric model.

IV. REQUIREMENT AND SOLUTIONS FOR THE HETEROGENEOUS CAD MODELING APPROACH

A CAD modeling approach for the components made of heterogeneous materials is developed. The task of this modeling system is to build the three types of sub-models respectively for each region, also to integrate them and then display both material and geometric information of any cross-section selected in a component. To meet these requirements, the functional requirements of the system can be decomposed into three sub-requirements, described as:

- $R_1$ = Building the three types of sub-models for each material region
- $R_2$ = Integrating these sub-models for each material region
- $R_3$ = Displaying both material and geometric information of any cross-section selected in the heterogeneous object

According to this modeling system, the solutions of these requirements must be chosen in such a way that each requirement can be satisfied by the adjustment of the corresponding solution without affecting other requirement. Hence, the system should be decomposed into three modules to satisfy the three sub-requirements, respectively as:

- $S_1$ = Develop a geometric and material module
- $S_2$ = Develop an integrating module
- $S_3$ = Develop an object demonstrating module

Accordingly, the relationship between requirements and solutions can be expressed by the following design Eqn.

$$\begin{align*}
R_1 &= \begin{bmatrix} X & 0 & 0 \end{bmatrix} S_1 \\
R_2 &= \begin{bmatrix} X & X & 0 \end{bmatrix} S_2 \\
R_3 &= \begin{bmatrix} X & X & X \end{bmatrix} S_3
\end{align*}$$

Fig. 1 Information flow of modeling and processing of the heterogeneous objects

Thus, the relationship between requirements and solutions can be expressed by the following design Eqn.

$$\begin{align*}
R_1 &= \begin{bmatrix} X & 0 & 0 \end{bmatrix} S_1 \\
R_2 &= \begin{bmatrix} X & X & 0 \end{bmatrix} S_2 \\
R_3 &= \begin{bmatrix} X & X & X \end{bmatrix} S_3
\end{align*}$$

where ‘X’ represents a non-zero element and 0 indicates zero elements. To complete the system design, the modules must be decomposed further for composition and micro structure of each material region.

V. MATHEMATICAL REPRESENTATION OF HETEROGENEOUS OBJECTS

In current work, mathematically the heterogeneous object is defined as:

$$P = (G, M) = (P_1, P_2, \ldots, P_n)$$

where $P_i$ represents the ith module of the heterogeneous object.
P = The heterogeneous object, made of n number of voxels, with geometric information G and material information M.

\[ P_i = (G_i, M_i) \]

where \( M_i = \sum_{j=1}^{k} x_j m_j \) and also \( \sum_{j=1}^{k} x_j = 1 \) for the \( i^{th} \) voxel with the geometric information \( G_i \) and material composition information \( M_i \) and also represents a row matrix, with its elements addressing the materials involved in a particular voxel. For homogeneous voxel, \( [c] = [m_t] \) where \( m_t \) is the \( t^{th} \) material from a list of total k number of predefined primary materials involved in HO and for a heterogeneous voxel consists of any three materials, \( [c] \) can be \([m_1, m_3, m_k]\). x in above equation represents the volume fraction of \( m_t \)th material.

\[ \sum_{i=1}^{n} G_i = P \text{ and } G_i = \sum_{j=1}^{v} G_{ij} \] (4)

For the heterogeneous object P, Eqn. 4 describes the complete geometric information of n number of voxels and v number of homogeneous unit volumes, where each \( i^{th} \) voxel is occupied by \( v \) number of homogeneous unit volumes thus results in a volumetric dataset of \( n \times v \) homogeneous unit volumes.

A. Material Distribution Function, \( f(s) \)

Material distribution function \( f(s) \) is a function of distance from the start of a functionally graded region to the end of same graded region. Either linear or non-linear mathematical function that fall on real domain can be used. The material distribution function \( f(s) \) with functionally graded region starting at the distance ‘a’ from the taken reference and ending with a distance ‘(1-a)’ is written as:

\[ f(s) = \begin{cases} 0, & s \leq a \\ f(s), & a < s < (1 - a) \\ 1, & s \geq (1 - a) \end{cases} \] (5)

The material distribution function and variation of different material’s composition in a graded region for two primary materials HO is shown in Fig 2 (a) and Fig. 2 (b) respectively.

B. Material Composition Arrays, \( (M_c) \) with Different Material Gradients

Each element of the material composition array \( M_c \) represents the volume fraction of pre-defined primary materials in \( G_i \) voxel and the total volume fraction of the primary materials for the material composition arrays should be summed up to one (described in Eqn. 3). Now for a selected region in HO, \( M_{cs} \) represents materials composition at the start and \( M_{cf} \) represents the materials composition at the end of selected gradient region. Thus the sum of respective volume fractions at the start and end of the gradient region has same relation as developed in Eqn. 3 i.e.

\[ \sum_{j=1}^{k} x_{cs} = 1 \text{ and } \sum_{j=1}^{k} x_{cf} = 1 \] (6)

where, \( x_{cs} = \) volume fraction of \( t^{th} \) material at the start of selected gradient region
\( x_{cf} = \) volume fraction of \( t^{th} \) material at the end of selected gradient region
\( k = \) number of primary materials including air

So, material composition for the above said selected gradient has the distribution described as:

\[ f(s) \times (M_{cs} - M_{cf}) + M_{cs}, \begin{cases} M_{cs} \in M \\ M_{cf} \in M \\ 0 \leq f(s) \leq 1 \end{cases} \] (7)

where \( M_{cs} \) and \( M_{cf} \) denotes the material composition at the start and end of the selected graded region and \( f(s) \) distribution of material composition in this region. Thus the Variation in material composition for a selected gradient region is shown in Fig. 2(b).

The property of heterogeneous voxel may be determined using Voigt’s rule as:

\[ S = \sum_{i=1}^{k} V_i S_i \] (8)

S is the resultant property of heterogeneous voxel
\( V_i \) is the total volume fraction of \( i^{th} \) material in unit volume or voxel
\( S_i \) is the property of \( i^{th} \) material.

For two materials composition, the heterogeneous property can be defined as:
\[ S = V_1 S_1 + (1 - V_1) S_2 \]  

(9)

Voxelization results in sharp material changes along the component boundaries, which potentially result in abrupt property (e.g. thermal expansion coefficient and stiffness) variations. So for the smooth material transition properties, one of the available blending functions may be used. In our case, the constant blending functions are incorporated at end positions and in between, a blending function, \( f_b \), along with the distance function is used to avoid the sharp change in material properties as shown in Fig.3.

\[ S = f(s)V_1 S_1 + (1 - V_1) f(s) f_b S_2 \]  

(10)

Fig. 3 Heterogeneous object with blended function

C. Complex Grading Regions

When two or more types of functional graded regions are associated with same geometric point, i.e. \( x \in D_1 \) & \( x \in D_2 \), where \( D_1 \) and \( D_2 \) denotes two functionally graded regions with different grading function, then they are tackled as similar to two types of loading on beam in mechanics with weighted ratio (\( w_1 \) and \( w_2 \)) and some priority tags. So, resulting operator \( \otimes \) is defined to sum up the effect of two types of grading as.

\[ D_1 \otimes w_1 w_2 D_2 = \frac{w_1 D_1 + w_2 D_2}{w_1 + w_2} \]  

(11)

The operators such as union, difference, intersection, immersion etc. are used with dominant material grading function information.

VI. A FRAMEWORK FOR HETEROGENEOUS OBJECT MODELING

The geometric model of HO has converted to voxels and further to volumetric datasets (VD) for the given 3D HO space. These volumetric datasets have the capability to represent the inner structures of an object such as material, color, density, and strength, which can be associated to respective voxel. Therefore, it is a perfect choice to utilize volumetric dataset to describe the internal properties or structures of a heterogeneous object. In our HO modeling scheme we take volumetric dataset as a carrier of homogeneous material primitives, while the shape of the object is described by the geometric model. These geometric and material models are flexible and efficient to evaluate different compositions. The geometry computation library offers lots of subroutines to process geometry computation. The main steps of framework for HO modeling are described in Fig.4 below:

![Fig.4 A framework for HO modeling system](image)

The basic data structure includes two main sub-modules called data processing and voxelization. The data processing module mainly copes with the data structure set up for geometric model having triangular meshes and subdividing the surfaces using repetitive loop structure while maintaining the sharp features of the object, thus results in smooth mesh boundaries between neighboring voxels. The geometric and material model are integrated to describe the shape and material information in a three dimensional HO space. As the volumetric dataset is a discrete representation of an object, the normal is lost in the process of voxelization. Thus, the rendered image of HO is not realistic. However, direct volume rendering (DVR) in scientific visualization is a powerful tool to render volumetric datasets. DVR is an approximate simulation of the propagation of light through a colored, semi-transparent gel where the color and opacity are functions of the scalar values in the volumetric dataset. The DVR algorithms fall into two categories, namely image based method and object based method, according to the methods of voxel projection. As described earlier, rapid prototyping technique offers a possibility to manufacture HO. The accuracy and quality of the final part fabricated by rapid prototyping depends on the 2D geometric slices of a model. The slices of the geometric model and the layers of the material volumetric dataset can be used to construct the 2D slices of heterogeneous object, which is called material resample with geometric constraint. The slicing algorithms are studied extensively in rapid prototyping community. There are mesh-based, direct, adaptive and hybrid slicing algorithms. In our framework, a mesh model slicing algorithm is developed based on directional weighted graph. At the end the dataset output files are displayed on the windows graphical user interface using many input-output functions and open graphical languages.

VII. CONCLUSION AND FUTURE SCOPE

This work presents a CAD modeling approach for heterogeneous object with simple geometry and complex material variations. The distribution of material is obtained by using the material distribution arrays and function for the distribution. The relation between
requirements of a heterogeneous CAD modeling system and their respective solution is established. The information flow chart will definitely provide the basic data structure to develop a new CAD model for heterogeneous objects. The framework stated in this paper for the fabrication of heterogeneous objects can be used as the hierarchy for the development of any CAD model using different approaches. The present work can be further extended and implemented on CAD software such as UNIGRAPHICS and CATIA which are capable of handling differential functions for complex and irregular HO. The approach can be extended to object modeling i.e. solid modeling with other physical attributes such as mechanical properties, material distribution etc. Dynamic heterogeneous objects (DHO) are the new class of heterogeneous objects. Unlike current heterogeneous object modeling, DHO deals with space dependent heterogeneities and time dependent material distributions. By taking time into consideration, more realistic process simulation can be achieved.

REFERENCES